

# **ON THE RECOVERY OF PEAK GROUND VELOCITY AND PEAK GROUND DISPLACEMENT FROM STRONG-MOTION RECORDS**

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## **ABSTRACT**

Peak ground velocity (PGV) and peak ground displacement (PGD) are important strong ground motion parameters for correlating with damage caused by an earthquake, for analysis of existing structures and for the design of new structures. Unlike peak ground acceleration, PGV and PGD are greatly affected by the accelerogram processing technique used. PGD is composed of two parts: the transient displacement which should equal zero at the end of the record and the displacement due to a permanent movement of the recording station, caused by fault slip or foundation failure for example. In the near field of large earthquakes permanent displacements can be much larger than transient displacements; however they are not recovered by standard processing methods based on filtering. Filtering of strong-motion records leads to underestimating PGV and PGD for large earthquakes which can have an important effect on work that employs these parameters.

*Keywords:* Strong ground motion, correction, peak ground velocity, PGV, peak ground displacement, PGD

## **INTRODUCTION**

Accelerograms, particularly those from analogue instruments, include long-period errors [e.g. 1] which can affect long-period strong-motion parameters such as PGV and PGD.

The major problem with the recovery of true ground velocity and displacement is that the zero acceleration level (baseline or centreline) is not indicated on the accelerogram [2]. The main difficulties in determining the baseline position are: a) initial part of shock is not recorded, b) final acceleration or velocity cannot be assumed to be zero, due to the presence of background noise and c) the final displacement is not known.

If the true baseline of the accelerogram can be estimated, hence minimizing long-period errors, then PGVs and PGDs recovered from the strong-motion record will be more physically reasonable than those obtained using current correction methods that employ filtering. This problem is addressed in this paper.

## IMPORTANCE OF PERMANENT GROUND DISPLACEMENT

Figure 1 shows  $M_w$  versus the maximum distance at which certain permanent displacements may be expected for vertical strike-slip faulting.

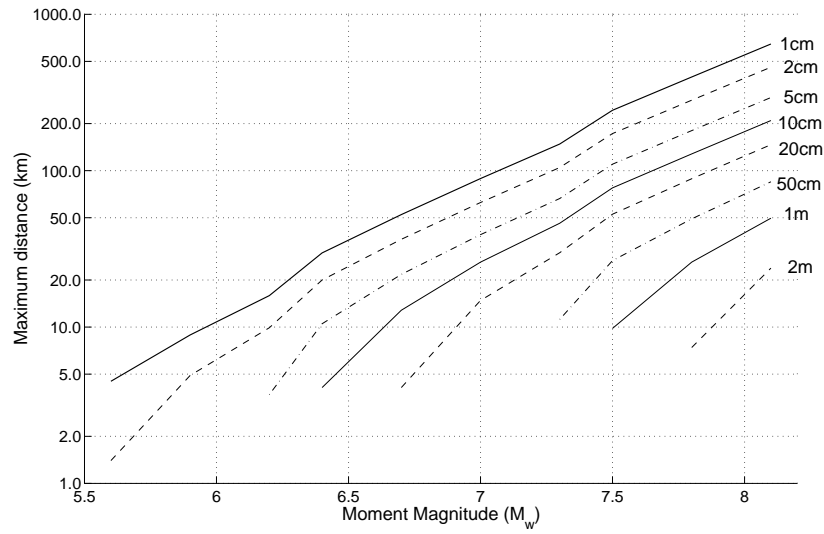


Figure 1: Moment magnitude,  $M_w$ , versus maximum distance from fault which undergoes a permanent deformation of 1 cm, 2 cm, 5 cm, 10 cm, 20 cm, 50 cm, 1 m or 2 m for vertical strike-slip faulting, along a perpendicular line to the fault, on the surface, from the centre of the rupture. The equations of Mansinha and Smylie [3] were used with the parameters: distance to top of fault,  $d = 0$  km, Wells and Coppersmith [4] equation for strike-slip faults:  $\log u = -6.32 + 0.90M_w$  where  $u$  is the average displacement along the fault, area of rupture,  $A = LW$  from:  $M_0 = \mu Au$  (where the rigidity of the crust  $\mu = 3 \times 10^{10}$  Nm) and  $W = L$  for  $L \leq 20$  km and  $W = 20$  km for  $L > 20$  km (assuming a depth of the seismogenic layer equal to 20 km).

From Figure 1 it may be seen that for large earthquakes permanent displacements are still large enough to be important for correct processing of strong-motion records even at considerable distances. For example for a  $M_w = 7.5$  earthquake deformations of about 1 m are to be expected at distances of up to 10 km. Therefore a technique that can recover these movements would lead to corrected records which more adequately represent the actual ground motion that occurred at the station.

## METHOD

The method chosen for this study is an extension of the method proposed by Graizer [5] based on the technique introduced by Hudson et al. [6]. Graizer [5] uses this technique to correct the 65° component of the Parkfield-Cholame Shandon Array 2W record from the Parkfield earthquake (28/6/1966) and Graizer [7] also uses it on the vertical accelerogram recorded at Karakyr during the Gazli earthquake (17/5/1976). For both records a good match is found with the measured and modelled permanent displacements that occurred from the earthquakes. The technique followed here is described below.

Let the digitised accelerogram be  $T$  seconds long. Firstly the uncorrected acceleration time-history is integrated once to get the uncorrected velocity,  $\dot{x}(t)$ . Next a polynomial,  $y(t)$ , which fits the beginning and the end of  $\dot{x}(t)$  is found by minimizing:

$$W = \int_0^{T_1} (\dot{x} - y)^2 dt + \int_{T_2}^T (\dot{x} - y)^2 dt; \quad (1)$$

where  $t = 0$  to  $t = T_1$  is the ‘quiet’ part of the record before the strong motion begins and  $t = T_2$  to  $t = T$  is the ‘quiet’ part after the strong motion has died out.  $T_1$  and  $T_2$  are chosen by firstly filtering the uncorrected time-history and then examining the graph of energy density versus time.  $T_1$  is chosen as the time just before the steepest part of the curve and  $T_2$  as the time just after the steepest part of the curve. The extended Graizer method is then applied with these choices and the displacement curves plotted. If the displacement curve is not reasonable then  $T_1$  and  $T_2$  are varied slightly until a better looking curve is found. However, a number of tests were made which showed that the permanent displacements and time-histories obtained are fairly insensitive to the choice of  $T_1$  and  $T_2$  [8], a similar finding to that of Graizer [5].

For a digitised record this minimization is equivalent to the well-known least-squares method. Once  $y(t)$  is found it is differentiated and this differentiated polynomial is subtracted from the *entire* acceleration time-history to find the corrected acceleration record. This is then integrated once to find the corrected velocity record and again to find the corrected displacement record.

The idea behind this technique is that the long-period noise in the record can be approximated by a polynomial and that it can be found using only the ‘quiet’ part of the record. This assumption means that the baseline found is not affected by the strong part of the record which may contain large velocity pulses which are likely to be present in near-field records [9].

For each accelerogram, as well as  $T_1$  and  $T_2$ , the degree of polynomial needs to be chosen. The accelerogram is first corrected with a linear polynomial (degree equals 1) and the acceleration, velocity and displacement judged against the criteria outlined below. If the correction is deemed not acceptable then the degree is increased by one, the correction applied again to the uncorrected accelerogram and the time-history examined. The smallest polynomial that yields an adequately corrected time-history is selected. This procedure partly runs in parallel to the procedure to select  $T_1$  and  $T_2$ .

Polynomials up to degree nine are used in this study; higher degree polynomials were found not to lead to more sensible corrected time-histories. All steps in the computer program were implemented using double precision arithmetic so that accurate results were obtained.

## TILTS

Many strong motion transducers are (or are equivalent to) pendulums that rotate due to acceleration of their supports [10]. Figure 2 is a diagram of one of these pendulums that measures horizontal translation acceleration  $\ddot{x}$ .

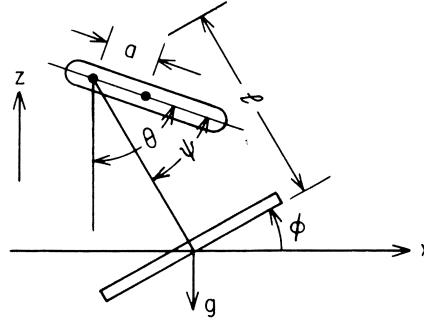


Figure 2: Schematic of strong-motion transducer pendulum making angle  $\psi$ , relative to its support frame, which in turn is tilted by angle  $\phi$ , with respect to the fixed gravity vector  $g$ . From Bradner and Reichle [11].

The equation of motion of the system in Figure 2 using the small angle approximation is [11]:

$$I\ddot{\theta} = -\mu a\dot{\psi} - ma g\theta + ma\ddot{x} + mal\ddot{\phi} \quad (2)$$

where  $I$  is moment of inertia,  $m$  is mass and  $\mu$  is viscous damping.

Now the horizontal deflection of the mass  $y = a\psi$  is measured by the instrument,  $\theta = \phi + \psi$  and letting  $2\xi\omega = \mu a/I$  and  $\omega^2 = ma g/I$  gives:

$$\ddot{y} + 2\xi\omega\dot{y} + \omega^2 y = \frac{a\omega^2}{g}\ddot{x} - a\omega^2\phi + a\left(\frac{l\omega^2}{g} - 1\right)\ddot{\phi} \quad (3)$$

The coefficients of horizontal translational acceleration,  $\ddot{x}$ , and the horizontal components of gravity caused by the tilt, approximately  $g\phi$ , are identical. Therefore the displacement,  $x$ , cannot be found by double integration of the output of the accelerometer [11]. Trifunac and Todorovska [10] have recently restated this although they use a slightly different equation of motion which includes cross-axis sensitivity. The equation of motion for instruments that measure vertical accelerations is not effected by tilts of the instrument [11]. If the effects of tilts on the recorded ground motion can be ignored then the recovery of the true ground displacement may be possible using an appropriate processing technique [5].

Tilts caused by the permanent ground deformation due to faulting can be estimated using equations predicting the static deformation of the ground from faulting. The tilts, due to the permanent ground deformation, are small [8] and consequently can possibly be neglected when correcting near-field accelerograms. However, in the near field the ground may exhibit non-linear behaviour resulting in large tilts; such tilts perhaps cannot be ignored.

## WAYS TO CHECK CORRECTION

One way of checking the corrected time-histories, although not always possible, is to compare the velocity and displacement curves obtained with simulated time-histories. Many papers [e.g. 12, 13] have calculated theoretical displacement traces for locations in the near field of large earthquakes against which the corrected displacement time-histories can be judged. Accelerograms recorded very close to the fault are influenced mainly by source mechanism rather than by inhomogeneities along travel path and consequently can be relatively simple. Therefore one criterion useful in the verification of the extended Graizer method is that the velocity and displacement time-histories are relatively simple, for example the velocity time-history could be composed mainly of a single large pulse which has been observed in both simulated and observed ground motions.

The corrected displacement time-histories should not show long-period waves present throughout the entire record which cannot be true ground motion because they do not begin at the beginning of the strong-motion portion of the record. The displacement should be roughly equal to zero for the part of the record before the strong-motion portion and then again constant in the part of the record after the strong-motion portion.

There are few studies which try to reproduce the recorded ground motions on near-field accelerograms to *include* the permanent displacement. Usually the computed ground motions are filtered to match the filtered accelerograms. Figure 3 shows the similarity between the corrected Pacoima Dam record from the San Fernando earthquake (9/2/1971) and the modelled displacements. Such a direct method of verification is often impossible, however, because modelling of many near-field accelerograms has not been done.

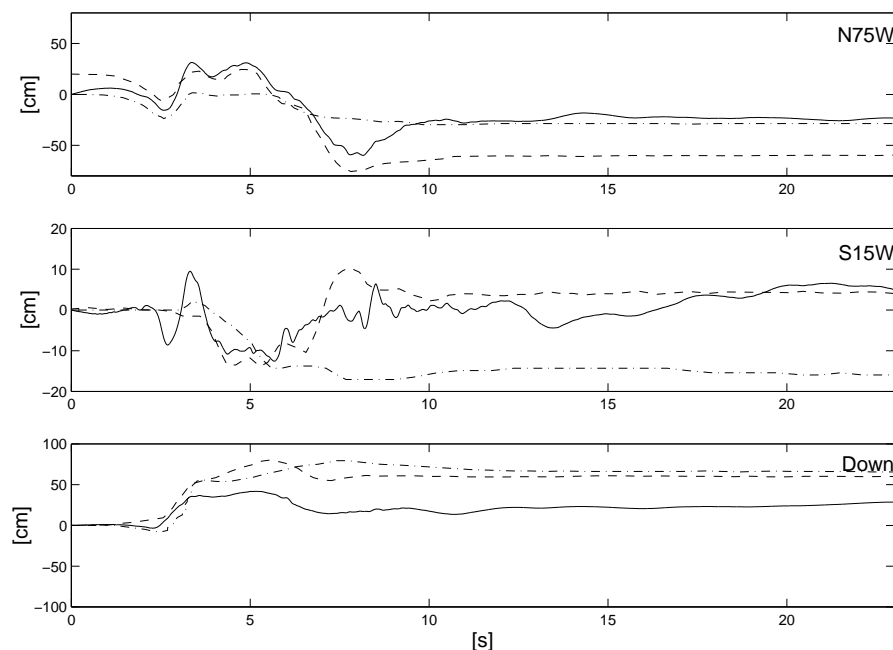


Figure 3: Comparison between corrected displacements of the Pacoima Dam record of the San Fernando earthquake (9/2/1971), using the extended Graizer method with the initial velocity not constrained to zero, and those modelled by Trifunac [14] and Heaton [15] for this station. The solid lines are the corrected displacements from the accelerogram and the dashed lines are the modelled displacements of Trifunac [14] (his Figure 6) and the dash-dotted lines are the modelled displacements of Heaton [15] (his Figure 10). The displacements of Trifunac [14] were shifted by 1.5 s so that they coincided with the displacements from the accelerogram.

Filtering techniques have been shown [16] to give accurate velocity and displacement time-histories, at least for a limited period range. Therefore if the velocity and displacement time-histories found using the extended Graizer method closely match those given by filtering then it is likely that the extended Graizer method yields reasonable corrected records for that accelerogram.

After a large earthquake measurements of the coseismic displacements (permanent displacements due to the earthquake) are now often made, using for example Global Positioning System (GPS) or Interferometric Synthetic Aperture Radar (InSAR). These measurements, if they exist, are extremely valuable in verifying the correction achieved by the extended Graizer method. If there are large differences between the measured coseismic displacement at a station and the recovered displacement from records at that station then the correction must be in error. Unfortunately measurements exist for only a few earthquakes and then the measurements are often not made exactly at the strong-motion stations thus differences between measured coseismic displacements and recovered displacements may be due to local effects at the station such as foundation failure.

Also the corrected velocities and displacements from stations which are close together should show similarities [17].

## PGV AND PGD FROM FILTERING

Figure 4 shows the displacement response spectra of three records, corrected using the extended Graizer method, from three earthquakes with increasing  $M_w$ . These figures show that spectral displacement (SD) becomes equal to PGD at a period,  $T_E$ , which is dependent on the magnitude of the earthquake, i.e. for small earthquakes SD becomes equal to PGD at periods much shorter than for large earthquakes. For periods greater than  $T_E$  SD is roughly constant and equal to PGD which shows that there is no significant energy in the corrected time-history at these longer periods. A similar period exists for spectral velocity (SV).

Liu and Helmberger [18] find that the Coyote Lake earthquake (6/8/1979) has a rupture length of 6 km and a rupture velocity of  $2.8 \text{ kms}^{-1}$ , which gives a rupture duration of about 2 s. Bouchon [19] calculates a rupture length of 14 km and a rupture velocity of  $2.6 \text{ kms}^{-1}$  and which gives a rupture duration of about 5 s. These rupture durations assume unilateral rupture. Figure 4 a shows that these calculated durations are roughly equal to the period at which SD becomes equal to PGD.

For the Imperial Valley earthquake (15/10/1979) Hartzell and Helmberger [20] find a rupture length of about 36 km and a rupture velocity between  $2.5$  and  $2.7 \text{ kms}^{-1}$  and hence a rupture duration of about 14 s and Archuleta [21] gives 37.5 km for the rupture length and between  $2.5$  and  $2.6 \text{ kms}^{-1}$  for the rupture velocity giving a rupture duration of about 15 s. These rupture durations assume unilateral rupture. As for the Coyote Lake earthquake the period at which SD becomes equal to PGD is close to the computed rupture duration of the Imperial Valley earthquake.

Yagi and Kikuchi [22] find the Chi-Chi earthquake (20/9/1999) has a rupture duration of about 32 s, which is again similar to the period at which SD becomes equal to PGD.

This finding is not completely new, for example Basili and Brady [23] note that it is important to use a low cut-off frequency less than the reciprocal of the length of strong-motion portion of the record which they note is roughly equivalent to the faulting duration and Trifunac [24] states that the corner frequency of source spectra is inversely proportional to rupture duration.

From this study of three near-field records it can be seen that an examination of the velocity and

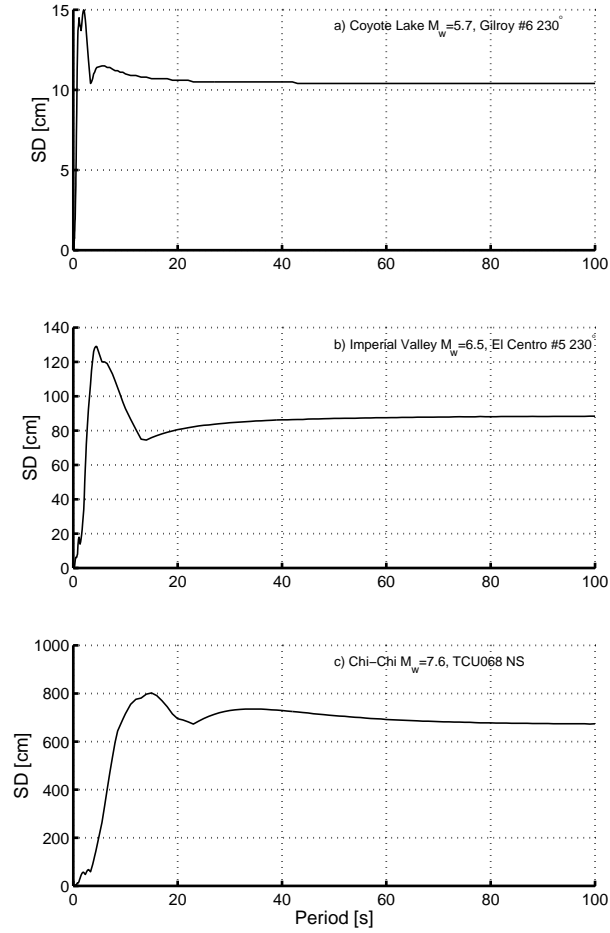


Figure 4: Comparison of displacement response spectrum, 5% damping, of a) Gilroy #6 230° record (Coyote Lake,  $M_w = 5.7$ ), b) El Centro #5 230° record (Imperial Valley,  $M_w = 6.5$ ) and c) TCU068 NS (Chi-Chi,  $M_w = 7.6$ ) from records corrected using extended Graizer method.

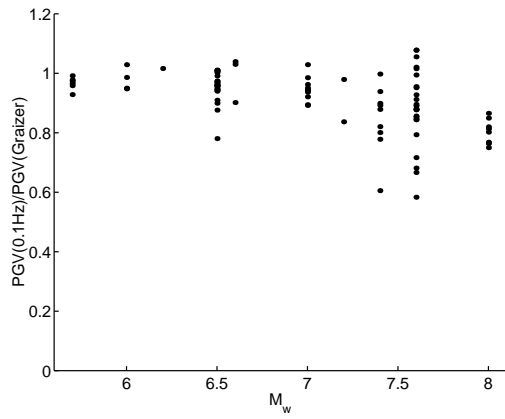
displacement response spectra of a record corrected using the extended Graizer algorithm is useful in deciding whether the correction achieved is reasonable. The correction procedure can be assumed to be adequate if the periods at which SV is roughly equal to PGV and SD roughly equals PGD is less than or equal to the rupture duration of the earthquake and that for longer periods the spectral ordinates are constant. If however, there is significant energy within the time-history for periods greater than the rupture duration, i.e. the SV and/or SD are not constant for periods greater than rupture duration, then this would mean that the correction made using the extended Graizer technique is probably incorrect.

Almost all strong-motion records are corrected using filtering techniques, the details of which vary but the results are similar: namely that the low-frequency (long-period) motion is removed. The low-frequency cut-offs which are employed are usually less than 0.1 Hz–0.2 Hz especially for acceleration time-histories from digital instruments. Filters are designed so that they have little effect on frequencies inside their pass-band and because the natural period of almost all engineering structures is less than about 4 s there should be little difference between frequency-domain parameters (such as response spectral values) calculated using different types of filter or different low-cut frequencies (as long as the low-cut frequency is below about 0.2 Hz). Time-

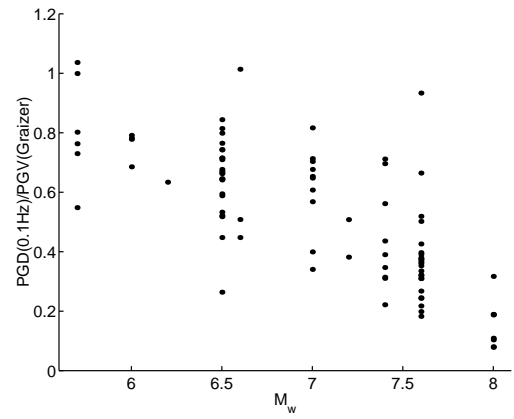
domain parameters such as peak ground velocity, PGV, and peak ground displacement, PGD, though can be greatly affected by differing correction procedures because such parameters are governed by a wide range of frequencies some of which may be altered by the correction procedure used.

It was found above that the period at which SV and SD becomes equal to PGV and PGD respectively is approximately equal to the rupture duration. Therefore using a low cut-off frequency which is greater than the reciprocal of the rupture duration will lead to the recovery of smaller velocities and displacements than actually occurred.

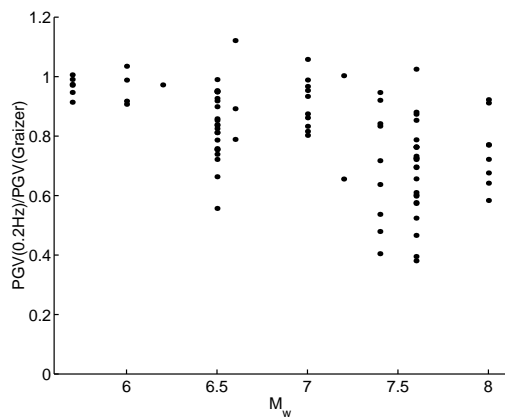
All of the records corrected using the extended Graizer method were filtered using an elliptical filter with low cut-off frequencies of 0.1 and 0.2 Hz. Figure 5 shows the ratio of PGV and PGD using the Graizer corrected records to PGV and PGD using the filtered records for two cut-off frequencies. It was assumed that all of the Graizer corrected records are realistic although some of the corrected displacements were probably incorrect. The main conclusion, however, is not likely to be strongly affected.



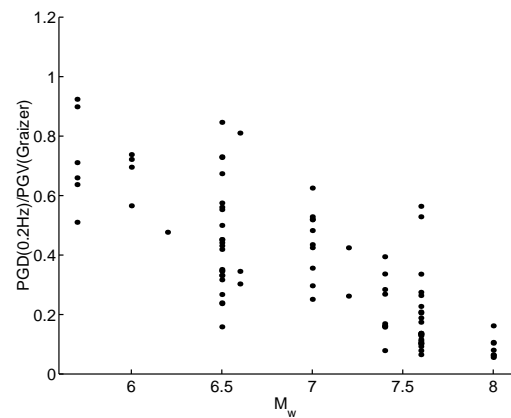
(a) PGV with low cut-off frequency of 0.1Hz



(b) PGD with low cut-off frequency of 0.1Hz



(c) PGV with low cut-off frequency of 0.2Hz



(d) PGD with low cut-off frequency of 0.2Hz

Figure 5: Ratio of PGV and PGD using a elliptical filter with low cut-off frequencies of 0.1 Hz and 0.2 Hz to PGV and PGD using the extended Graizer method with the initial velocity constrained to zero. The largest horizontal component was used.

Figure 5 shows that PGV is underestimated for records from earthquakes with  $M_w > 7$  using a low



cut-off frequency of 0.1 Hz and it is underestimated for records from earthquakes with  $M_w > 6.5$  using a low cut-off frequency of 0.2 Hz. Therefore use of filtering techniques, with low cut-off frequencies 0.1–0.2 Hz, for the correction of accelerograms from earthquakes with  $M_w < 6.5$  will not significantly underestimate PGV. However, for near-field accelerograms from earthquakes with  $M_w > 6.5$  filtering with standard low cut-off frequencies of 0.1–0.2 Hz will lead to significant underestimation of PGV; for such records either the low cut-off frequencies needs to be reduced or baseline correction procedures, like that investigated here, adopted.

Figure 5 shows that PGD is greatly underestimated for records from earthquakes with  $M_w > 6$  using a low cut-off frequency of 0.1 or 0.2 Hz and that the size of the underestimation increases with magnitude. For records from earthquakes with  $M_w = 8$  the PGD recovered from the strong-motion records using a filter with a low cut-off frequency of 0.2 Hz is about ten times smaller than that recovered using the extended Graizer method. Part of the reason why filtering near-field records yields much smaller PGD than the extended Graizer method is that filtering does not recover the permanent ground displacement, which, as has been shown, is large and increases with magnitude. Therefore if permanent displacement is significant at a strong-motion station, filtering will lead to much lower estimates of PGD than would a baseline correction procedure, like that used in this study.

The permanent offset observed from an earthquake generally takes several seconds to reach its full extent therefore it is doubtful that it is important for buildings. Thus the transient part of the ground motion that occurred during the earthquake may be a more useful measure of the displacement for engineering design. However, a consistent and useful definition of transient peak ground displacement is difficult to find because PGD is strongly affected by the low-frequency cut-off and also because it is difficult to separate the transient and permanent ground displacement.

Details of the correction method used, the sensitivity of the method and the results obtained from numerous near-field strong-motion records from 16 earthquakes can be found in Douglas [8].

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